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Effect of Initial Stress on a Fiber-Reinforced Anisotropic Thermoelastic Thick Plate

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Abstract The two-dimensional problem of generalized thermoelasticity for a fiber-reinforced anisotropic thick plate under initial stress is studied in the context of the Lord and Shulman theory. The upper surface of the plate is thermally insulated with prescribed surface loading while the lower surface of the plate rests on a rigid foundation and temperature. The problem is solved numerically using a finite element method. Numerical results for the temperature distribution, and the displacement and stress components are given and illustrated graphically. It is found from the graphs that the initial stress significantly influences the variations of field quantities. The results obtained in this paper may offer a theoretical basis and meaningful suggestions for the design of various fiber-reinforced anisotropic thermoelastic elements under loading to meet special engineering requirements.

Keywords Fiber-reinforced · Finite element method · Generalized thermoelastic · Initial stress · Thick plate

List of Symbols

ρ Mass density
σ_0 Initial stress
t_0 Relaxation time

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$\lambda, \mu_{\mathrm{T}}$	Elastic parameters
u, v	Displacement components
$\sigma_{xx}, \sigma_{xy}, \sigma_{yy}$	Stress components
c_{e}	Specific heat at constant strain
T	Temperature change of a material particle
T_0	Reference uniform temperature of the body
β_{11}, β_{22}	Thermal elastic coupling components
K_{11}, K_{22}	Thermal conductivity components
$\alpha, \beta, (\mu_{\mathrm{L}} - \mu_{\mathrm{T}})$	Reinforced elastic parameters
$\alpha_{11},\ \alpha_{22}$	Coefficients of linear thermal expansion

1 Introduction

Fiber-reinforced materials have many applications in aerospace and automotive fields, as well as in sailboats, and notably in modern bicycles and motorcycles, where their high strength-to-weight ratio is of importance. Improved manufacturing techniques are reducing the costs and time to manufacture, making it increasingly common in small consumer goods as well, such as laptops, tripods, fishing rods, paintball equipment, archery equipment, racquet frames, stringed instrument bodies, and classical guitar strings. The mechanical behavior of many fiber-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fiber direction. In such composites the fibers are usually arranged in parallel straight lines. Sengupta and Nath [1] discussed the problem of surface waves in fiber-reinforced anisotropic elastic media. Singh [2] showed that, for wave propagation in fiber-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. Hashin and Rosen [3] gave the elastic moduli for fiber-reinforced materials. In the classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible. Lord and Shulman [4] introduced a theory of generalized thermoelasticity with one relaxation time for an isotropic body. The theory was extended for an anisotropic body by Dhaliwal and Sherief [5]. In this theory, a modified law of heat conduction including both the heat flux and its time derivatives replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both coupled and uncoupled theories of thermoelasticity. Singh [6] studied wave propagation in thermally conducting linear fiber-reinforced composite materials with one relaxation time. Verma [7] discussed the problem of magnetoelastic shear waves in self-reinforced bodies. Chattopadhyay and Choudhury [8] investigated the propagation, reflection, and transmission of magnetoelastic shear waves in self-reinforced media. Chattopadhyay and Choudhury [9] studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. Chattopadhyay and Michel [10] studied a model for spherical SH-wave propagation in self-reinforced linearly elastic media. Tian et al. [11], Abbas

[12], Abbas and Abd-Alla [13], Abbas and Othman [14], and Youssef and Abbas [15] applied the finite element method in different generalized thermoelastic problems. Because of the importance of the initial stresses in applications, studies on the analysis of such effects have been carried out with different objectives and approaches [16–18].

In the present contribution, the two-dimensional problem of generalized thermoelasticity for a fiber-reinforced anisotropic thick plate under initial stress is studied. The problem has been solved using generalized thermoelasticity theory proposed by Lord and Shulman. The governing equations for displacement and temperature fields are solved by a finite element method. Numerical results for the temperature distribution, and the displacement and stress components are given and illustrated graphically. The results obtained in this paper may offer a theoretical basis and meaningful suggestions for the design of various fiber-reinforced anisotropic thermoelastic elements under loading to meet special engineering requirements.

2 Formulation of the Problem

We consider an infinite homogeneous fiber-reinforced anisotropic thermoelastic thick plate. Let the faces of the plate be the planes $x = \pm l$, referred to a rectangular set of Cartesian co-ordinates axes ox, oy, and oz as shown in Fig. 1.

We shall consider two-dimensional deformation of the plate parallel to the xy plane. Then the components of the displacement vector and temperature can be taken in the following form:

$$u = u_x = u(x, y, t), v = u_y = v(x, y, t), w = u_z = 0, T = T(x, y, t). \qquad (1)$$

Following Lord and Shulman [4], Singh [6], and Qian et al. [16], the constitutive relations and field equations with an initial stress and without body forces and heat

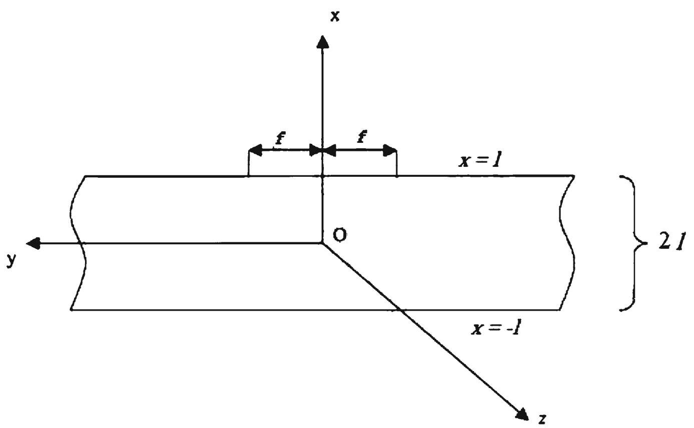

Fig. 1 Co-ordinate system and geometry of the plate